WOPA-Porto-2017
Workshop on Orthogonal Polynomials and Applications

Analysis Area - Center of Mathematics of University of Porto (CMUP)

Mathematics Department of Faculty of Sciences of University of Porto

Porto, September 5th, 2017

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Support:

- Center of Mathematics of University of Porto CMUP (UID/MAT/00144/2013) funded by FCT (Portugal) with national (MEC) and European structural funds (FEDER), under the partnership agreement PT2020.

- Mathematics Department of Faculty of Sciences of University of Porto

The **WOPA-Porto-2017, Workshop on Orthogonal Polynomials and Applications**, is a one day meeting organized by Analysis Area of Center of Mathematics of University of Porto (CMUP) and will be held at Mathematics Department of Faculty of Sciences of University of Porto.

This event takes place following the past four similar meetings held in Porto in 2003, 2006, 2007 and 2016, that gathered PhD and Post-Doc students and their supervisors, members of CMUP and some researchers from other universities of Portugal and also from other countries, working in this field of research. This year, we want to continue the same principle in a workshop we intend to make annual.

We would like to thank all the speakers who make this event possible, in special Pascal Maroni, who visits Portugal every year by sharing with us his latest research.

We hope this workshop will be pleasant and profitable for all participants.

The organizer,

Zélia da Rocha

Venue: FC1 Maths Building

Place: Room FC1.007

Shedule: Tuesday, September 5th

9h 30m	**Registration**	
9h 50m	**Opening Session**	
10h	Pascal *Maroni*	CNRS, Univ. Paris VI, France
11h	Youssèf *Ben Cheikh*	Univ. Monastir, Tunisia
12h	José C. *Petronilho*	CMUC, FCTUC, Coimbra
12h 30m	**Lunch**	
14h 30m	Semyon *Yakubovich*	CMUP, DM-FCUP, Porto
15h	Ana Filipa *Loureiro*	Univ. Kent, United Kingdom
15h 30m	Kenier *Castillo*	CMUC, Coimbra
16h	Ângela *Macedo*	UTAD, Vila Real
16h 30m	**Coffee break**	
17h	José *Matos*	CMUP, ISEP, Porto
17h 30m	Marco *Martins Afonso*	CMUP, Porto
18h	Helder *Lima*	Univ. Kent, United Kingdom
18h 30m	Zélia *da Rocha*	CMUP, DM-FCUP, Porto
19h	**Closing**	

Programme

- **9h 30m Registration**

- **9h 50m Opening Session**

- **10h - 11h**
  Pascal Maroni, CNRS, University of Pierre Marie Curie - Paris VI, France
  Convolution and Fourier transforms around vector spaces of polynomials functions
- **11h - 12h**
  Youssèf Ben Cheikh, University of Monastir, Tunisia
  Some partitions of polynomials sets
- **12h - 12h 30m**
  José Carlos Petronilho, CMUC, University of Coimbra
  On inverse problems and semi-classical orthogonal polynomials

- **12h 30m - 14h 30m Lunch**

- **14h 30m - 15h**
  Semyon Yakubovich, CMUP, University of Porto
  Certain classes of the index transforms and higher order PDE
- **15h - 15h 30m**
  Ana Filipa Loureiro, University of Kent, United Kingdom
  Spectral approximation of convolution operator
- **15h 30m - 16h**
  Kenier Castillo, CMUC, Coimbra
  On variation of zeros of paraorthogonal polynomials on the unit circle
- **16h - 16h 30m**
  Ângela Macedo, University of Trás-os-Montes e Alto Douro, Vila Real
  Symbolic approach to the general quadratic polynomial decomposition

- **16h30m - 17h Coffee Break**

- **17h - 17h30m**

  José Matos, CMUP, School of Engineering, Polytechnic Institute of Porto

  Stabilizing numerical operations with orthogonal polynomials

- **17h30m - 18h**

  Marco Martins Afonso, CMUP, Porto

  Applications of multivariate Hermite polynomials in fluid dynamics

- **18h - 18h30m**

  Helder Lima, University of Kent, United Kingdom

  On Müntz-type formulas related to the Riemann zeta function

- **18h30m - 19h**

  Zélia da Rocha, CMUP, University of Porto

  Connection coefficients, zeros and interceptions points of perturbed Chebyshev polynomials

- ## 19h Closing

Abstracts

- *Convolution and Fourier transform around the vector space of polynomial functions*

Pascal Maroni

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University of Pierre Marie Curie - PARIS VI - France

Abstract:

The principal aim of this talk is to place the space of all polynomial functions, $\mathcal{P}$, and its dual, $\mathcal{P}'$, among other usual spaces such that:

a) the space of all real (complex) valued indefinitely differentiable functions of compact support on $]-\infty,+\infty[$, $\mathcal{D}$, and its dual, $\mathcal{D}'$;

b) the space of all real (complex) valued indefinitely differentiable functions on $\mathbb{R}$, $\mathcal{E}$ and its dual $\mathcal{E}'$;

c) the space of all real (complex) valued indefinitely differentiable functions on $\mathbb{R}$ with the property that, for all integers $m \geq 0$ and $n \geq 0$, $|t|^n|\varphi^{(m)}(t)| \to 0$ as $|t| \to \infty$, $\mathcal{S}$, and its dual , $\mathcal{S}'$;

and so on ....

The used method is essentially a walking in the landscape around $\mathcal{P}$ and $\mathcal{P}'$ with the aid of the concept of convolution and the Fourier transform. The using of duality is intensive.

- *Some partitions of polynomials sets*

Youssèf Ben Cheikh

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Abstract:

Let $\mathbb{P}$ be the set of polynomial sequences $\{P_n\}_{n\geq 0}$, where $P_n \in \mathbb{C}[X]$ and degree(P_n) $= n$. Polynomial sequences are a topic of interest in many branches of mathematics as algebraic combinatorics, enumerative combinatorics, approximation theory and statistics. The literature contains many examples and classes of polynomial sequences.

In this work, we consider three sets of operators acting on formal power series:

• $\Lambda^{(-1)}$ the space of linear operators λ acting on formal power series that reduce the degree of every polynomial by exactly one and $\lambda(1) = 0$.

• $\vee^{(-1)}$ the space of linear operators acting on formal power series that reduce the order of every formal power series f, $f(0) = 0$, by exactly one.

• $\mathcal{S}_0$ the set of all power series $A(t) = \sum_{n=0}^{\infty} a_n t^n$, $a_0 \neq 0$. That may also be considered as a set of operators acting on formal power series, the set of multiplicative operators by A.

We show that any partition of these three sets leads to a partition of $\mathbb{P}$. Then we define suitable partitions of these sets to obtain the origins of some well known classes in $\mathbb{P}$. We propose some models for the description. Such partitions were used to treat various problems related to polynomial sets, we survey some of them.

- *On inverse problems and semiclassical orthogonal polynomials*

José Carlos Petronilho

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Abstract:

In this talk we consider inverse problems in the theory of orthogonal polynomials (OP), related with the notion of (generalized) coherent pair of measures and also with polynomial mappings in the framework of semiclassical OP

Thus, on the one hand, we analyze a structure relation of the type

$$\sum_{i=0}^{N} r_{i,n}\, Q_{n-i+m}^{(m)}(x) = \sum_{i=0}^{M} s_{i,n} P_{n-i+k}^{(k)}(x)\ , \quad n = 0, 1, 2, \dots\ ,$$

where $(P_n)_n$ and $(Q_n)_n$ are given OP sequences, the orders of the derivatives, k and m, being arbitrarily fixed nonnegative integer numbers, and $r_{i,n}$ and $s_{i,n}$ are complex numbers fulfilling some appropriate conditions. Such structure relation leads to the notion of $(M, N)-$coherence of order (m, k), being an extension of the concept of coherence of measures (as introduced by A. Iserles, P. E. Koch, S. P. Nørsett, and J. M. Sanz-Serna). We will show how these notion can be useful in approximation theory involving Sobolev OP. The appropriate framework for the study of such relations is the theory of semiclassical orthogonal polynomials.

On the other hand, we analyze sequences of monic OP $\{p_n\}_{n\geq 0}$ and $\{q_n\}_{n\geq 0}$ linked by a polynomial mapping, in the sense that there exist polynomials π_k and θ_m, of degrees k and m, respectively, being $0 \leq m \leq k-1$, such that

$$p_{nk+m}(x) = \theta_m(x)\, q_n(\pi_k(x))\,, \quad n = 0, 1, 2, \dots\ .$$

We will point out several applications (e.g., connections with sieved OP, OP on the unit circle, spectral theory of Jacobi operators, and Linear Algebra). In particular, the positive-definite case leads to orthogonality measures with support on several intervals. We will consider the special case whether one of the sequences $\{p_n\}_{n\geq 0}$ or $\{q_n\}_{n\geq 0}$ is semiclassical, recovering in an unified way recent results obtained by several authors for some particular polynomials mappings.

References

[1] K. Castillo, M.N. de Jesus, and J. Petronilho, *On semiclassical orthogonal polynomials via polynomial mappings*, *J. Math. Anal. Appl.* (2017). In press.

[2] M.N. de Jesus, F. Marcellán, J. Petronilho, N.C. Pinzón-Cortés, $(M, N)-$coherent pairs of order (m, k) and Sobolev orthogonal polynomials, *J. Comput. Appl. Math.* **256** (2014) 16-35.

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- *Certain classes of the index transforms and higher order PDE*

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Abstract:

We discuss new index transforms, which are associated with the modified Bessel functions as the kernel. The boundedness and invertibility are examined for these operators in the Lebesgue weighted spaces. Inversion theorems are proved. Important particular cases are exhibited. The results are applied to solve boundary value problems for higher order PDE, involving the Laplacian.

In particular, the main objects of the present talk are the index transforms, containing as the kernel products of the modified Bessel functions. Precisely, we will consider the following operator and its adjoint

$$(Ff)_\alpha(\tau) = \frac{2\sqrt{\pi}}{\cosh(\pi\tau)} \int_0^\infty \mathrm{Re}\left[K_{\alpha+i\tau}(\sqrt{x})\ I_{\alpha-i\tau}(\sqrt{x})\right] f(x)dx,\ \tau \in \mathbb{R},$$

$$(Gg)_\alpha(x) = 2\sqrt{\pi} \int_{-\infty}^\infty \mathrm{Re}\left[K_{\alpha+i\tau}(\sqrt{x})\ I_{\alpha-i\tau}(\sqrt{x})\right] \frac{g(\tau)}{\cosh(\pi\tau)} d\tau,\ x \in \mathbb{R}_+,$$

where $\alpha \in \mathbb{R}$ is a parameter, i is the imaginary unit and Re denotes the real part of the complex-valued function and $K_\mu(z), I_\mu(z)$ are modified Bessel functions.

References

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- *Spectral approximation of convolution operator*

Ana Filipa Loureiro

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This is a joint work with **Kuan Xu**

Abstract:

Convolution operator is ubiquitously dense in mathematics and engineering. While approximations via classic orthogonal polynomials for many commonly-used operators, e.g. integration and differentiation, are well-known for decades and become indispensable in approximation theory and spectral methods, spectral approximation of convolution operator hasnt been attempted until very recent years. In this talk, some recent results on the convolution of classic orthogonal polynomials and spectral approximations of convolution operator will be presented. These results enable accurate computation of convolution integrals and are believed to lay the foundation of the spectral methods for convolution integral equations.

- *On variation of zeros of paraorthogonal polynomials on the unit circle*

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Abstract:

Readers familiar with the literature on orthogonal polynomials known that the dramatic difference between *orthogonal polynomials on the real line* and *orthogonal polynomials on the unit circle* is channeled by *paraorthogonal polynomials on the unit circle* (hereafter abbreviated by POPUC). The behavior of their zeros, directly or indirectly, is the main reason by which POPUC have received significant attention over the last years. From the theoretical point of view, POPUC answer a problem posed by Turán at the beginning of the 1970's [3, Problem LXVI, p. 60]: *"It is known that the zeros of the nth orthogonal polynomial (with respect to a Lebesgue-integral function on an interval) separate the zeros of the* $(n+1)$*th polynomial. What corresponds to this fact on the unit circle?"* [1]. As far as we can tell, this question was solved accidentally by Delsarte and Genin [2] when they were working in linear prediction theory. After that, several authors stated additional properties of zeros of POPUC. A recent work with refined results on the interlacing of zeros of POPUC and historical comments can be found in [2]. It is well known that POPUC can be regarded as the characteristic polynomials of any matrix similar to a unitary upper Hessenberg matrix with positive subdiagonal elements. The purpose of this talk is to establish —using in a consequent manner basic methods of linear algebra—, in terms of the primary coefficients in the framework of the tridiagonal theory developed by Delsarte and Genin in the environment of nonnegative definite Toeplitz matrices, necessary and sufficient conditions for the monotonicity with respect to a real parameter of eigenvalues of unitary upper Hessenberg matrices with positive subdiagonal elements. Further results within the broader context of matrices with simple eigenvalues on the unit circle are also discussed. All the results that will be presented in this talk are included in [1].

References

[1] K. Castillo. On monotonicity of zeros of paraorthogonal polynomials on the unit circle. Technical report, arXiv:1706.05709, 2017.

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[3] P. Turán. Some open problems in approximation theory (in Hungarian). *Mat. Lapok*, 25:21-75, 1974.

[1]We quote the English translation provided by Szüsz.

[2]These authors never mentioned the connection with the question posed by Turán.

- *Symbolic approach to the general quadratic polynomial decomposition*

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This is a joint work with **Z. da Rocha** *and* **T.A. Mesquita**

Abstract:

In this work, we deal with a symbolic approach to the general quadratic polynomial decomposition, in the sequel to what was done in [6]. By means of a symbolic implementation, we investigate some properties of the components sequences like orthogonality and symmetry. We present some explicit results for a collection of well known orthogonal cases.

References and Literature for Further Reading

[1] P. Maroni, Sur la decomposition quadratique d'une suite de polynômes orthogonaux. I., *Rivista di Matematica Pura ed Aplicata*, N.6 (1990) pp. 19-53.

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- *Stabilizing numerical operations with orthogonal polynomials*

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This is a joint work with **João C. Matos, M. João Rodrigues, Marcelo Trindade** *and* **Paulo B. Vasconcelos**

Abstract:

In this work we introduce a set of formulas suitable to build algebraic and differential operations on polynomials in the form of matrix operations. These operational matrices' elements can be evaluated with a mix process, combining explicit and recursive formulas. When implemented with spectral methods, these processes improve significantly the solution's accuracy. In some cases, the spectral rate of convergence can be recovered.

Problem and Method

If we are interested in solving an integro-differential problem $Dy = f$, getting the solution in the form of a generalized Fourier series $y = a\mathcal{P}$ and if the problem has smooth conditions, then spectral methods are an excellent choice, which are known to have spectral order of convergence. Nevertheless, the numerical implementation deals with operations in orthogonal polynomials, which are often unstable. This serious drawback can retard the convergence rate or even prevent the convergence of the method.

An usual way of implementing a spectral method is to translate the action of the integro-differential operator D on Fourier series into algebraic operations in the coefficients vector a, reducing the problem to the solution of a linear system $Ma = f$. An approximate solution $y_n = a_n\mathcal{P}_n$ is obtained solving an adequate truncated system $M_n a_n = f_n$. A numerical problem arises when the condition number of M_n is an increasing function of n. To prevent losing convergence, two problems must be tackled with care: choose an adequate algebraic solver and evaluate M carefully.

When P is an orthogonal polynomials sequence, we introduce a combination of explicit and recursive formulas to evaluate the coefficients of M. Our process produces more accurate and better conditioned matrices, improving the accuracy in the spectral solution.

- *Applications of multivariate Hermite polynomials in fluid dynamics*

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Abstract:

In 1865 Charles Hermite introduced the multivariate version of his orthogonal polynomials, suitable for a multidimensional problem with a Gaussian weight in the Ornstein–Uhlenbeck formalism. Such tensorial polynomials have often been used in theoretical physics to describe the quantum harmonic oscillator, by means of Hermitianization process and second quantization algorithm, introducing the vacuum state and the ladder (creation/annihilation) operators. We will show an interesting application of this framework to – classical — fluid mechanics, in order to obtain the transport properties of particles carried by a flow field. Namely, we study the sedimentation, diffusion and dispersion of inertial particles, endowed with a small-but-finite radius and a mass density different from the surrounding environment. We are going to present results from existing scientific literature [1,2,3] and from ongoing work (M. Martins Afonso *et al.*, 2017, under submission).

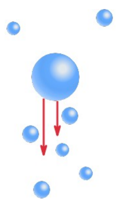

Figure 1: Droplet collision and coalescence represent the last and essential stage of rain formation. This process is driven by the difference in settling between drops of different sizes, a phenomenon that can be attacked analytically by making use of the multivariate Hermite polynomials.

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- *On Müntz-type formulas related to the Riemann zeta function*

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Abstract:

The talk is based on the paper available at `arxiv.org/abs/1705.09386`

We start by presenting the Mellin integral transform (and its inverse) [1], exhibiting several Dirichlet series related with the Riemann zeta function [2] and introducing a new family of classes of functions that generalises the Mntz-type class of functions defined in [4].

We use the Mellin transform and the Dirichlet series mentioned above to obtain integral transformations with arithmetic functions (see [3]) in the half-plane $Re(s) > 1$ and, as a result of moving those integrals to the left, we derive, for the classes of functions mentioned above, what we call Müntz-type formulas, which are identities similar to the classical formula

$$\zeta(s)\int_0^\infty f(y)y^{s-1}dy = \int_0^\infty \left(\sum_{n=1}^\infty f(nx) - \frac{1}{x}\int_0^\infty f(t)dt\right)x^{s-1}dx$$

deduced originally by Müntz and valid in the critical strip $0 < Re(s) < 1$ (see [2]). We derive several Müntz-type formulas not only in the strip $0 < Re(s) < 1$ but also in strips across the half-plane $Re(s) < 0$.

Finally, we exhibit representations in form of Mellin transforms for products of the gamma and zeta functions, as particular cases of the Müntz-type formulas previously obtained.

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- *On connection coefficients, zeros and interception point of perturbed Chebyshev polynomials*

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Abstract:

Orthogonal polynomials satisfy a recurrence relation of order two, where appear two coefficients. If we modify one of these coefficients at a certain order, we obtain a perturbed orthogonal sequence. In this work we consider in this way some perturbed of Chebyshev polynomials of second kind and we deal with the problem of finding the connection coefficients that allow to write the perturbed sequence in terms of the original one and in terms of the canonical basis. From the connection relations obtained and from two other relations, we deduce some results about zeros and interception points of these perturbed polynomials. All the work is valid for arbitrary order of perturbation.

References

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