The generalized Bochner condition about classical orthogonal polynomials revisited

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Abstract

We bring a new proof for showing that an orthogonal polynomial sequence is classical if and only if any of its polynomial fulfils a certain differential equation of order $2k$, for some $k \geqslant 1$. So, we build those differential equations explicitly. If $k = 1$, we get the Bochner's characterization of classical polynomials. With help of the formal computations made in *Mathematica*, we explicitly give those differential equations for $k = 1, 2$ and 3 for each family of the classical polynomials. Higher order differential equations can be obtained similarly.

Key words: Classical orthogonal polynomials, classical forms, Bochner's differential equation
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