Learning by replicator and best-response: the importance of being indifferent

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Abstract

This paper compares two learning processes, namely those generated by replicator and best-response dynamics, from the point of view of the asymptotics of play. We base our study on the intersection of the basins of attraction of locally stable pure Nash equilibria for replicator and best-response dynamics. Local stability implies that the basin of attraction has positive measure but there are examples where the intersection of the basin of attraction for replicator and best-response dynamics is arbitrarily small. We provide conditions, involving the existence of an unstable interior Nash equilibrium, for the basins of attraction of *any* locally stable pure Nash equilibrium under replicator and best-response dynamics to intersect in a set of positive measure. Hence, for any choice of initial conditions in sets of positive measure, if a pure Nash equilibrium is locally stable, the outcome of learning under either procedure coincides. We provide examples illustrating the above, including some for which the basins of attraction exactly coincide for both learning dynamics. In the case of total coincidence of the basins of attraction of the stable Nash equilibria, we show the role that indifference sets play in this feature.

Keywords: best-response dynamics; replicator dynamics; learning; basin of attraction