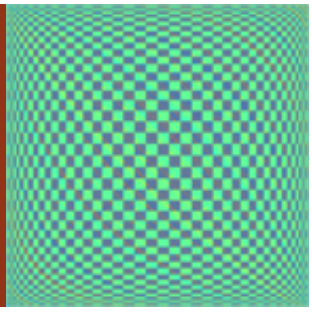
Centro de **Matemática**
Universidade do Porto



Seminar on Semigroups, Automata and Languages

Structure of Jeandel-Rao aperiodic Wang tilings

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Abstract: Wang tilings is an important class of tilings of the plane where each tile (called Wang tile) is a unit square with a color given to each edge. Tilings by Wang tiles are valid when the colors of the common edge of every pair of adjacent tiles within the tiling are the same. In 1961, Wang conjectured that when a finite set of Wang tiles admit a valid tiling of the plane, it must admit one which is periodic. This statement was proved false few years later by Berger, a student of Wang, who also provided a set of 20426 (lowered down later to 104) Wang tiles that admit valid tilings the plane none of them being periodic. Such set of Wang tiles are called *aperiodic*. In the 1970s, Penrose found an aperiodic set of two rhombus with matching rules, and their apparition in aluminium-manganese alloys was first noticed by crystallographers in the 1980s (leading to a Nobel Prize in Chemistry in 2011). A question which was open for a long time was to find the smallest set of aperiodic Wang tiles. Before 2015, the smallest aperiodic sets of Wang tiles known were Kari's 14 tiles and Culik's 13 tiles both discovered in 1996. Then in 2015, Jeandel and Rao closed the question and proved the existence of an aperiodic set of 11 Wang tiles and that no set of Wang tiles of cardinality ≤ 10 is aperiodic. Their computation of all candidates of size up to 11 "took approximatively one year on several hundred cores".

In this talk, we will discuss the structure of Jeandel-Rao aperiodic tilings admitted by the 11 Wang tiles discovered by Jeandel and Rao with wooden laser-cut puzzle pieces. The most important result being the existence of a polygonal partition of the two-dimensional torus and a $\mathbb{Z}^2$-action on the torus allowing to easily construct valid Jeandel-Rao tilings. This can also be seen as the projection of points from a higher dimensional lattice as it was already noticed by de Bruijn in 1981 in the context of Penrose tilings. Time will not allow to discuss other results related to the higher dimensional extensions of Sturmian sequences, Markov partition, Rauzy induction, but the curious reader can find them in the three following articles published in 2021: `https://doi.org/10.1007/s00454-019-00153-3`, `https://doi.org/10.5802/ahl.73`, `https://doi.org/10.3934/jmd.2021017`

Date: Friday, 16 December 2022, 14:30
Place: Online Zoom meeting

Com o apoio do projeto UIDB/00144/2020