Centro de **Matemática**
Universidade do Porto

Seminar on
Semigroups, Automata and Languages

Simplicity of Nekrashevych algebras of contracting self-similar groups

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Abstract: A self-similar group is a group G acting on the Cayley graph of a finitely generated free monoid X^* (i.e., regular rooted tree) by automorphisms in such a way that the self-similariy of the tree is reflected in the group. The most common examples are generated by the states of a finite automaton (or more precisely transducer). Many famous groups like Grigorchuk's 2-group of internediate growth are of this form.

Associated to each self-similar group is a left cancellative monoid $M = G \cdot X^*$ generated by the actions of G and X^* on the left of X^*. The inverse hull of M is a congruence-free inverse monoid generalizing the polycyclic inverse monoid (which arises in the case G is trivial).

Nekrashevych associated C^*-algebras and algebras with coefficients in a field to self-similar groups. These are tight algebras of the associated inverse monoid in the sense of Exel. In the case G is trivial, the algebra is the classical Leavitt algebra, a famous finitely presented simple algebra.

Nekrashevych showed the algebra of the Grigorchuk group is not simple in characteristic 2, but Clark, Exel, Pardo, Sims and Starling showed its Nekrashevych algebra is simple over all other fields. Nekrashevych then showed that the algebra of the Grigorchuk-Erschler group is not simple over any field (the first such example).

The Grigorchuk and Grigorchuk-Erschler groups are contracting self-similar groups. This important class of self-similar groups includes Gupta-Sidki p-groups and many important iterated monodromy groups like the Basilica group. Nekrashevych proved algebras of contacting groups are finitely presented.

In this talk we discuss a recent result of the speaker and N. Szakacs (York/Szeged) characterizing simplicity of Nekrashevych algebras of contracting groups. In particular, we give an algorithm for deciding simplicity given an automaton generating the group. We apply our results to several families of contracting groups like Gupta-Sidki groups and Sunic's generalization of Grigorchuk's group.

Date: Friday, 25 September 2020, 14:30
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