CENTRO DE MATEMÁTICA UNIVERSIDADE DO PORTO

A brief introduction to representation theory of quivers and cluster-tilting theory

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Abstract

The main objective of representation theory is the study of algebras via the study of their modules (i.e. representations). In this mini-course we will consider a combinatorial approach to this theory, using quivers (i.e. oriented graphs) and their representations (given by vector spaces associated to vertices and linear maps associated to arrows). This approach can be used to study the representation theory of all finite-dimensional algebras, since these can be realised as quotients of path algebras of a quiver.

Schedule—all sessions held in room 1.20/1.21

day	time
Monday, May 15th	11:00-13:00
Tuesday, May 16th	13:30-15:30
Wednesday, May 17th	11:00-13:00
Thursday, May 18th	11:30-13:00
Friday, May 19th	13:30-15:00

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Outline of the course

I. A brief introduction to Auslander-Reiten (AR) theory, culminating in the definition and computation of AR-quivers. The AR-quiver of a finite-dimensional algebra encodes the structure of its module category, in the sense that vertices correspond to indecomposable modules and arrows correspond to 'irreducible' morphisms.

Summary of content:

- Quivers, representations of quivers, morphisms of representations;
- Path algebra of a (bound) quiver;
- Basic definitions of categories and functors;
- Equivalence of categories: representations of quivers versus modules;
- Short exact sequences, projective and injective modules/representations; projective and injective resolutions; AR-translation; almost split sequences;
- Extensions and Ext;
- AR-quiver: Knitting algorithm.
- Gabriel's theorem: classification of algebras with finitely many indecomposable modules (up to isomorphism).

II. An introduction to recent topics of research in the area, namely cluster-tilting theory. This theory simultaneously provides a 'categorification' of the combinatorics of cluster algebras and a generalisation of (APR-)tilting theory, which is an important technique enabling the comparison of the representation theories of different algebras.

Summary of content:

- APR-tilting; BGP-reflections; APR-tilted algebras;
- Derived categories and cluster categories;
- Cluster-tilting objects; and cluster-tilted algebras.

Given the time constraints, no proofs will be given, but we will illustrate and motivate the theory using several examples.