

## Algebra, Combinatorics and Number Theory Seminar

**Date.** Friday, July 04, 2025 - 11.30am Porto - FC1 007 (and online) <sup>1</sup>

**Speaker.** Rustam Turdibaev - New Uzbekistan University

**Title.** GIT quotients of some matrix pairs

**Abstract.** The classification of tuples of matrices up to simultaneous conjugation is a classical problem in linear algebra and representation theory. While the case of a single matrix leads to the well-known Jordan normal form, the situation becomes far more intricate for tuples, where the problem is considered "wild" and therefore not classifiable in any reasonable sense. Geometric Invariant Theory (GIT) provides a powerful framework for studying such spaces through quotient varieties, which parametrize semisimple representations of free associative algebras.

In this talk, after briefly reviewing our recent results on invariants of  $4 \times 4$  matrix pairs - with applications to the invariant commuting variety and the fourth Calogero-Moser space - we turn our focus to the GIT quotient of the space of anti-commuting matrix pairs. This variety exhibits a rich and highly reducible geometric structure and differs in significant ways from commuting variety.

We study the algebra of invariants on the anti-commuting variety, identify a general generating set - proven to be minimal for matrix sizes up to five - and develop an algorithmic method to derive the defining relations among these generators. Using computational tools such as Macaulay2, we verify these relations case by case, thereby providing a complete description of the invariant ring for matrices of size up to five.

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<sup>1</sup><https://fc-up-pt.zoom.us/j/85243807260>